GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(AUTONOMOUS)

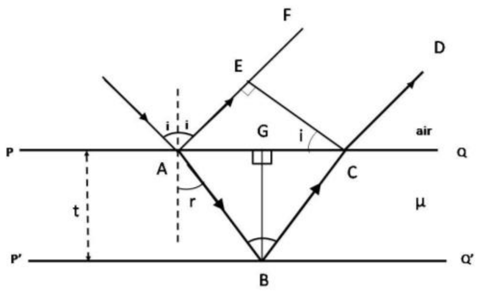
(Affiliated to Andhra University, Visakhapatnam)

I B.Tech. - II Semester Regular Examinations, June/July – 2025

ENGINEERING PHYSICS KEY

1. **a. Explain interference in thin films in reflected light and build an expression for maxima condition and minima condition.**

Let us consider PQ and P'Q' are the two surfaces of a transparent film of uniform thickness t and refractive index μ as shown in figure. Let a ray of monochromatic light of wave length λ is incident on its upper surface at an angle i. This ray is partly reflected along AF and partly refracted along AB. After one reflection at B the ray moves in the direction of BC. After refraction at C, the ray finally emerges out along CD in air. As the rays AF and CD are derived from same source, therefore they are coherent. The find the path difference between the rays AF and CD, draw a normal CE on to AF.



The effective path difference is given by (AB+BC) in film – (AE) in air

$$\delta = \mu\,(AB+BC) - AE \qquad (1)$$

The angle ∠BGA and ∠BGC are equal to ∠r, therefore AB = BC, substituting this condition in equation (1)

The effective path difference $\delta = 2\mu.AB - AE$

From the right angled triangle ΔBGA, $\cos r = \frac{BG}{AB}$

$$\therefore AB = \frac{BG}{\cos r} = \frac{t}{\cos r}$$

From the right angled triangle ΔBGA, $\sin r = \frac{AG}{AB}$ $\therefore AG = AB \sin r$

From the right angled triangle ΔAEC, $\sin i = \frac{AE}{AC}$

$$\therefore AE = AC \sin i = 2AB \sin r \sin i$$

since AC= 2AG=2AB sin r

Therefore, AE = $\frac{2t}{\cos r}$ sin i sin r $= \frac{2\mu t}{\cos r}$ $\sin^2 r$ (since sin i/ sin r = μ)

Thus the path difference between the two reflected rays :

$$\delta = 2.\frac{\mu t}{\cos r} - 2.\frac{\mu t}{\cos r}.\sin^2 r$$

$$\delta = 2.\frac{\mu t}{\cos r}[1-\sin^2 r]$$

$$\delta = 2\mu t.\cos r$$

The above expression is known as "cosine law". It is clear from the above figure that the wave along AF reflected from denser medium, hence there occurs a phase change of π or path difference $\frac{\lambda}{2}$

∴ The additional path difference is $\frac{\lambda}{2}$

∴ The effective path difference $\delta = \mathbf{2\mu t \cos r} + \frac{\lambda}{2}$

Condition for maxima and minima

For maxima the path difference should be equal to nλ

$$\therefore 2\mu t \cos r + \frac{\lambda}{2} = n\lambda \text{ or}$$

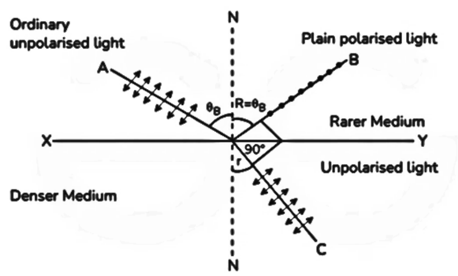
$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \qquad \text{where } n= 1, 2, 3\ldots$$

For minima the path difference should be equal to $(2n+1)\frac{\lambda}{2}$

$$\therefore 2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \qquad \text{where } n= 0, 1, 2, 3\ldots$$

b. **State and Explain Brewster's Law.**



Brewster's Law states that when unpolarized light strikes an interface between two transparent dielectric media, the reflected light will be **completely plane-polarized** if the reflected ray and the refracted (transmitted) ray are **perpendicular** to each other. The angle of incidence at which this occurs is called **Brewster's Angle** or the **Polarizing Angle**, denoted as θp.

The relationship between Brewster's Angle (θp) and the refractive indices of the two media is given by the formula:

$$\tan(\theta p) = n1/n2$$

Where:

- θp is Brewster's Angle (the angle of incidence for complete polarization).
- n1 is the refractive index of the first medium (where the light is coming from).
- n2 is the refractive index of the second medium (the material the light is entering).

Often, light is assumed to be traveling from air (n1≈1) into another medium, simplifying the formula to:

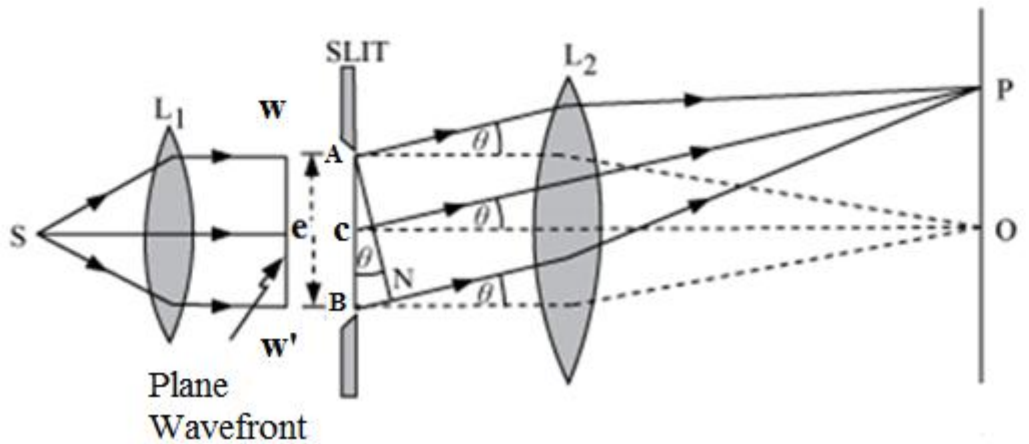
$$\tan(\theta p) = n$$

Where n is the refractive index of the second medium.

2. a. Derive an expression for resultant intensity due to single slit diffraction.

Consider a slit AB of width **e** perpendicular to the plane of the paper. Let a plane wave front WW' of monochromatic light of wave length **λ** propagating normally to the slit is incident on AB.

- According to Huygens wave theory every point on the wave front incident on the slit is act as a source of secondary wavefronts.
- These secondary wavefronts travelling normal to the slit along 'CO' are brought to focus at O by using lens L_2 on the screen.
- The secondary wavelets travelling at an angle θ with normal are focused at a point P on the screen.

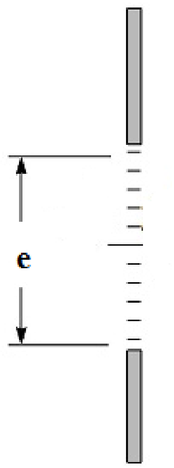


The intensity at these points (O and P) depends on the path difference between secondary waves originating from the slit AB. Since the secondary wavefronts, which are travel normal to the slit have no path difference, therefore, the intensity at point O is maximum and is known as "Central Maximum".

To find the intensity at P, draw a normal AN on BP. The path difference between extreme rays from slit AB is

$$BN = AB \sin\theta = e \sin\theta$$

$$\therefore \text{ The phase difference } = \frac{2\pi}{\lambda}\,(e\ \sin\theta)$$



Let the slit AB is divided into large number of n equal parts and the phase difference between any two consecutive parts is equal to

$$\frac{1}{n}\left[\frac{2\pi}{\lambda}(e\ \sin\theta)\right] = d \ (\text{say})$$

The Resultant amplitude at P due to secondary waves from each slit having amplitude **a** and phase difference **d** between successive waves is given by

$$R = a\,\frac{\sin\left(\frac{nd}{2}\right)}{\sin\frac{d}{2}}$$

$$= a\,\frac{\sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin\theta}{n\lambda}\right)}$$

$$= a\,\frac{\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)} \qquad \text{where } \alpha = \frac{\pi e \sin\theta}{\lambda}$$

Since n is large when compared with $\frac{\alpha}{n}$, $\sin\left(\frac{\alpha}{n}\right)$ may be replaced by $\frac{\alpha}{n}$

$$R = a\,\frac{\sin\alpha}{\left(\frac{\alpha}{n}\right)} = n\,a\left(\frac{\sin\alpha}{\alpha}\right)$$

$$= A\left(\frac{\sin\alpha}{\alpha}\right) \quad (\text{where } A = n\,a)$$

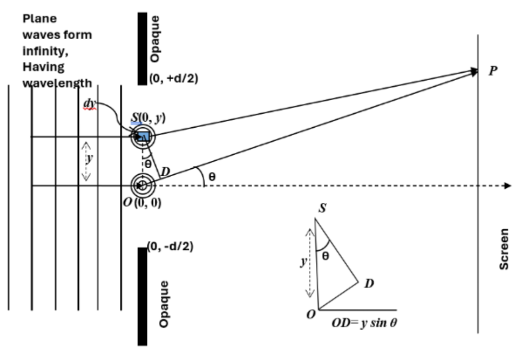
$\therefore$ The Resultant intensity at point 'P' on the screen is

$$I = A^2\left(\frac{\sin^2\alpha}{\alpha^2}\right) \qquad , \text{where} \quad \alpha = \frac{\pi e \sin\theta}{\lambda}$$

(OR)

Integration Method is also valid

Consider a rectangular slit of width ***d*** extending along y – axis as shown below. Choose the center of this slit as origin with coordinates O (0, 0). The extent of slit is from (0, -d/2) to (0,+d/2).



Let a Plane parallel beam of rays (plane wavefronts) of wavelength λ fall on the slit from left hand side as shown in the figure. Let there be a distant screen with point of observation P making an angle *θ* with horizontal as shown in the figure. We are interested in calculating the intensity at P due the overlap of all rays (waves) coming from the entire slit.

According to Huygens theory of light every point of the wavefront is a source of secondary spherical wavelets. Let there be a point *S* (0, y) where there is a point spherical source with width *dy* as shown. The wave reaching at P from an arbitrary point *S* (0, *y*) can described as

$$d\psi_P = (a\ dy).e^{i\varphi}$$

Where a is the amplitude of received light (plane wave) per unit width of the slit, φ is the phase of the spherical wave reaching at P coming from S. The amplitude of spherical wave at a location of y having finite width of dy will be $a.dy$ .The phase φ can be calculated as

$$\varphi = \frac{2\pi}{\lambda}.path\ (\Delta)$$

Path Δ will be the path difference created between the wave coming directly from the center of the slit at O and the ray coming from S, which is OD from the figure.

$$\varphi = \frac{2\pi}{\lambda}.OD$$

$$\varphi = \frac{2\pi}{\lambda}.y\sin\theta$$

(as OD = $y\ sin\theta$ from the Δ ODS)

$$\varphi = qy \quad with\ q = \frac{2\pi}{\lambda}.\sin\theta$$

Finally the dy contributes to point P with $\boldsymbol{d\psi_P}$ given by

$$\boldsymbol{d\psi_P = (a\ dy).e^{i\varphi} = a\ dy\ e^{iqy}} \qquad \text{………….. (1)}$$

Where, a is the amplitude per unit width of the slit

dy is the elemental point source located at y units distance from origin on the slit

$$i = \sqrt{-1}$$

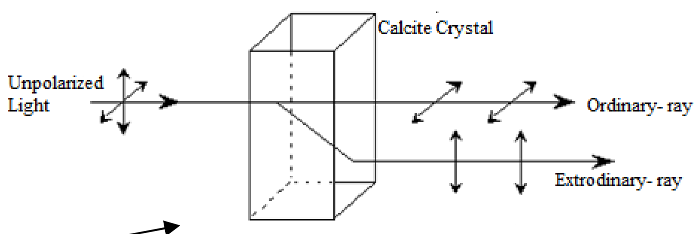
$$q = \frac{2\pi}{\lambda}.\sin\theta$$

θ = angle made by point P at origin

The following figure shows the point spherical sources sending rays that overlap at P on the screen. There are only 8 spherical sources shown, but in reality there will be infinitely many such spherical point sources.

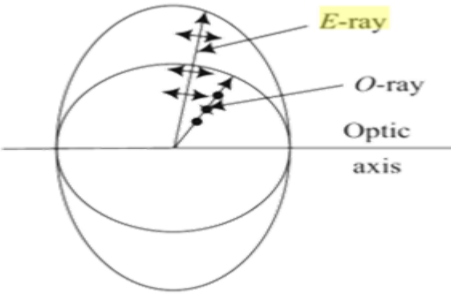
2. b. Summarize the phenomenon of double refraction in calcite crystal.

Double refraction, also called as **birefringence**. It is an optical property in which a single ray of unpolarized light entering an anisotropic medium is split into two rays, each traveling in a different direction.
Consider a Calcite crystal, when an unpolarized light incident on this crystal, the refracted ray split in to two plane polarized rays.

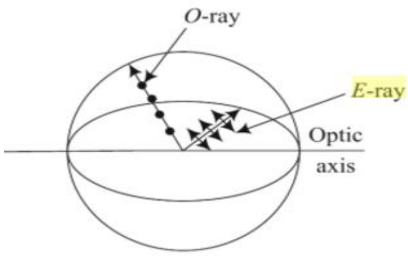


- One ray (calle inary ray: E-ray) is bent, or refracted, at an angle as it travels through the medium; the other ray (called the ordinary ray: O-ray) passes through the medium unchanged.

- Furthermore, the refractive index of the O-ray is observed to be constant in all directions; the refractive index of the E-ray varies with respect to the direction.
- Therefore, O-ray travels with same speed in all directions whereas E-ray travels with different speeds along different directions.
- Thus, an E-ray can move either faster or slower than an O-ray.
- However, the velocity of E-ray and O-ray are equal when they travel along the optic axis direction.
- If the velocity of E-ray is more than that of O-ray then the crystal is said to be negative crystal (ex: Calcite)
- If the velocity of E-ray is less than that of the O-ray then the crystal is said to be positive crystal (ex: Quartz)



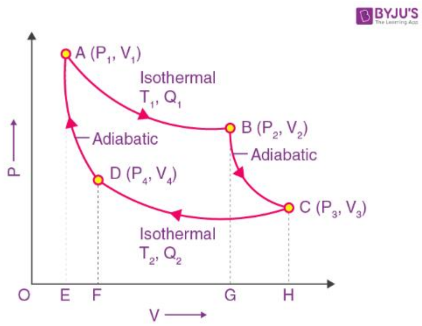
(a) Negative crystal



(b) Positive crystal

3. a. What is Carnot's cycle? Build an equation for the efficiency of Carnot's heat engine.

A Carnot cycle is defined as an ideal reversible closed thermodynamic cycle in which there are four successive operations involved, which are isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression.



Following are the four processes of the Carnot cycle:

• In (a), the process is reversible isothermal gas expansion. In this process, the amount of heat absorbed by the ideal gas is qin from the heat source, which is at a temperature of Th. The gas expands and does work on the surroundings.

Isothermal expansion: The gas is taken from P_1, V_1, T_1 to P_2, V_2, T_2. Heat Q_1 is absorbed from the reservoir at temperature T_1. Since the expansion is isothermal, the total change in internal energy is zero, and the heat absorbed by the gas is equal to the work done by the gas on the environment, which is given as:

$$W_{1\rightarrow 2} = Q1 = \mu \times R \times T_1 \times ln\frac{v2}{v1} \qquad ...(1)$$

• In (b), the process is reversible adiabatic gas expansion. Here, the system is thermally insulated, and the gas continues to expand and work is done on the surroundings. Now the temperature is lower, Tl.

Adiabatic expansion: The gas expands adiabatically from P_2, V_2, T_1 to P_3, V_3, T_2.

Here, work done by the gas is given by:

$$W_{2\to3} = \frac{\mu R}{\gamma-1}(T_1 - T_2) \qquad ...(2)$$

• In (c), the process is reversible isothermal gas compression process. Here, the heat loss qout occurs when the surroundings do the work at temperature Tl.

Isothermal compression: The gas is compressed isothermally from the state (P_3, V_3, T_2) to (P_4, V_4, T_2).

Here, the work done on the gas by the environment is given by:

$$W_{3\to4} = \mu R T_2 \ln\frac{v3}{v4} \qquad ...(3)$$

• In (d), the process is reversible adiabatic gas compression. Again the system is thermally insulated. The temperature again rises back to Th as the surrounding continue to do their work on the gas.

Adiabatic compression: The gas is compressed adiabatically from the state (P_4, V_4, T_2) to (P_1, V_1, T_1).

Here, the work done on the gas by the environment is given by:

$$W_{4\to1} = \frac{\mu R}{\gamma-1}(T_1 - T_2) \qquad ...(4)$$

Hence, the total work done by the gas on the environment in one complete cycle is given by:

$$W = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1}$$

Putting the values of works from above work equations 1 to 4

$$W = \mu R T_1 \ln\frac{v2}{v1} - \mu R T_2 \ln\frac{v3}{v4}$$

$$Net\ efficiency = \frac{Net\ work\ done\ by\ the\ gas}{Heat\ absorbed\ by\ the\ gas}$$

$$Net\ efficiency = \frac{W}{Q1} = \frac{Q1-Q2}{Q1} = 1 - \frac{Q2}{Q1} = 1 - \frac{T2}{T1}\frac{\ln\frac{v3}{v4}}{\ln\frac{v2}{v1}}$$

Since the step 2→3 is an adiabatic process, we can write $T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$

Or, $V_2 / V_3 = (T_2/T_1)^{1/\gamma-1}$

Similarly, for the process 4→1, we can write

$$V_1 / V_4 = (T_2 / T_1)^{1/\gamma-1}$$

This implies, $$V_2 / V_3 = V_1 / V_4$$

$$V_2 / V_1 = V_3 / V_4$$

So, the expression for net efficiency of carnot engine reduces to:

$$Net\ efficiency\ of\ Carnot\ heat\ engine = 1 - \frac{T2}{T1}$$

3. b. Analyse the relation between Entropy and Second Law of Thermodynamics.

Entropy:

The concept of entropy refers to state of order represented by S.

A change in order is a change in the number of ways of arranging the particles, and it is a key factor in determining the direction of any process.

Solid $\longrightarrow$ Liquid $\longrightarrow$ Gas

More Order $\longrightarrow$ Less Order

The increase in entropy is given by dS = $\frac{dQ}{T}$.

dS value depends only on the initial and final states of the system

Absorption of heat increases entropy of the system. In a reversible adiabatic change dq=0, the entropy change is zero

The net entropy change in a reversible process is zero,

Second Law of Thermodynamics:

Second law of thermodynamics is a fundamental law of nature which explains that heat can flow only from hot body to cold body by itself. There are several statements of this law. Two are the most significant viz.,

(a) Kelvin- Planck statement:

No process is possible whose sole result is the absorption of heat from a source and the complete conversion of the heat into work.

It is impossible to get a continuous supply of work from a body by cooling it to a temperature lower than that of the surroundings.

(b) Clausius statement:

No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.

It is impossible for self-acting machine to transfer heat from colder body to hotter body without an aid of external agency.

Relation:

- The Second Law of Thermodynamics also states that entropy of the entire universe, as an isolated system, will always increase over time.
- The second law also states that the changes in the entropy in the universe can never be negative.
- The entropy of an isolated system increases in the course of a spontaneous change: $\Delta S_{tot} > 0$. where S_{tot} is the total entropy of the overall isolated system.

4. a. What is an adiabatic process? Derive an expression for the work done in adiabatic process.

Adiabatic Process:

A process in which a system undergoes physical changes in such a way that the total heat energy remains constant hence heat energy neither allowed to enter the system from the surrounding nor allowed to leave the system to the surroundings is called an adiabatic process.

(i) In adiabatic process, the total heat energy remains constant hence $\Delta Q = 0$

(ii) From the first law of thermodynamics $dU + dW = 0$ or $dU = -dW$.

(iii) In this process PV^{γ} = constant (or) $TV^{\gamma-1}$ = constant (or) $P^{1-\gamma} T^{\gamma}$ = constant

Work done during adiabatic process:

Consider one mole of an ideal gas enclosed in an adiabatic chamber i.e. the walls of chamber; base and piston are bad conductors. Now the gas expands at constant heat energy from volume V_1 to V_2. Let the corresponding pressure be P_1 and P_2

At any instant let "P" be the pressure of the gas, the motion of the piston through an elementary change dx, area of the piston is "A" then the work-done is given by

For an adiabatic change

$PV^{\gamma} = K$ (constant) $\qquad \therefore P = K/V^{\gamma}$

$$\therefore W = \int_{V_1}^{V_2} \frac{K}{V^{\gamma}} . dV = K\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$\therefore W = \frac{K}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

$$\text{(Or) } W = \frac{1}{1-\gamma}\left[KV_2^{1-\gamma} - KV_1^{1-\gamma}\right]$$

But $P_1V_1^{\gamma} = P_2V_2^{\gamma} = K$

$$\therefore W = \frac{1}{1-\gamma}\left[P_2V_2^{\gamma}V_2^{1-\gamma} - P_1V_1^{\gamma}V_1^{1-\gamma}\right]$$

$$W = \frac{1}{1-\gamma}[P_2V_2 - P_1V_1]$$

$$\therefore W = \frac{1}{1-\gamma}[RT_2 - RT_1]$$

$$\therefore W = \frac{R}{1-\gamma}[T_2 - T_1]$$

$$\boxed{\therefore W = \frac{R}{\gamma-1}[T_1 - T_2]}$$

This expression gives the work-done for a 1 mole of gas.

For 'n' moles of gas

$$\therefore W = \frac{nR}{\gamma-1}[T_1 - T_2]$$

4. b. Find the efficiency of Carnot's engine working between steam and ice points

The efficiency of a Carnot heat engine is given by: $\eta_c=1-T_H/T_L$

Where:

- **Ice Point (T_L):** 0∘C=0+273.15 K=273.15 K
- **Steam Point (T^H):** 100∘C=100+273.15 K=373.15 K

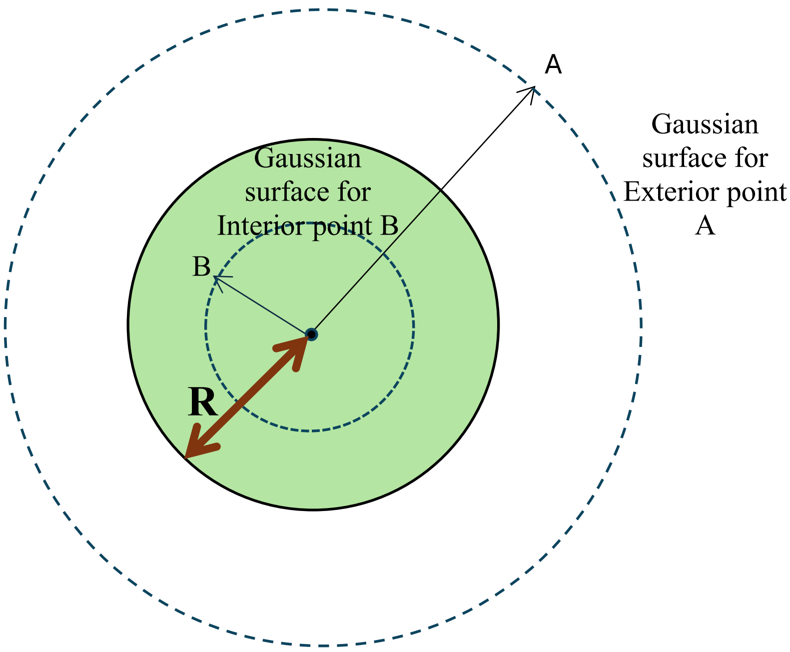
Now, substitute these values into the efficiency formula: ηc==1−373.15 K/273.15 K

ηc= 1−0.7319

ηc= 0.2681

The maximum possible efficiency of a Carnot engine working between the steam and ice points is approximately **26.81%**.

5. a. Applying Gauss's law of electrostatics, develop an expression for the electric field due to uniformly charged sphere at a point (i) outside the sphere and (ii) inside the sphere.



Consider a Spherical Charge distribution of Radius R as shown (shaded sphere) above. Let there be a uniform charge of magnitude Q distributed over the sphere with charge density ρ. $\left\{\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right\}$.

We are interested in calculating the electric field due to this charge distribution at three different points near the sphere.

1. Exterior point A: A is at a distance of "r" from the center of this charge distribution (r>R)
2. Interior point B: B is at a distance of "r" (r<R) from the center of this charge distribution.

Case1: At A, Let us construct a Gaussian sphere centered at the same center as that for the charge distribution with radius r(>R) shown as a dotted sphere (outer) in the above diagram. Applying Gauss law to this sphere gives,

$$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\epsilon_o}\{net\ charge\ enclosed\ by\ the\ Gaussian\ surface\}$$

The total charge enclosed by outer sphere with radius "r" (>R) is Q; $\left\{\rho\frac{4}{3}\pi R^3 = Q\right\}$. As the magnitude of Electric field E on the surface of this sphere with radius "r" remains constant (though its direction will definitely change from point to point on the same sphere), E can be taken outside of the integration. The angle between tiny element $\overrightarrow{dS}$ and $\vec{E}$ is always zero degree as the radius vector of sphere is always parallel to the Electric field at the point on the surface of sphere.

$$\oiint E\ dS \cos 0 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho\frac{4}{3}\pi R^3\right\}$$

$$E \oiint\ dS\ \times\ 1 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho\frac{4}{3}\pi R^3\right\}$$

$$\oiint\ dS\ = surface\ area\ of\ sphere\ with\ radius\ (r > R) = 4\pi r^2$$

$$E\ 4\pi r^2 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho\frac{4}{3}\pi R^3\right\}$$

$$E(r > R)\ = \frac{Q}{4\pi r^2\epsilon_o} = \frac{1}{4\pi r^2\epsilon_o}\left\{\rho\frac{4}{3}\pi R^3\right\} = \left\{\frac{\rho R^3}{3r^2\epsilon_o}\right\}$$

Case2: For interior point at C, a similar application like above holds. Construct a Gaussian sphere with radius r < R from the same center as that of charge distribution. For the application of Gauss law, the total charge contained within this Gaussian sphere needs to be calculated. For the above two cases, the Gaussian surfaces hold same quantity of charge Q, but in this case the total charge enclosed by the smaller sphere is not the whole charge Q but a portion of it.

$$Q_{enclosed}\ =\ \rho\frac{4}{3}\pi r^3$$

$$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\epsilon_o}\{Q_{enclosed}\} = \frac{1}{\epsilon_o}\left\{\rho\frac{4}{3}\pi r^3\right\}$$

As for this Gaussian sphere too the similar argument holds and the left hand side of the above equation transforms into

$$\oiint E\, dS \cos 0 = E\, 4\pi r^2$$

$$E\, 4\pi r^2 = \frac{1}{\epsilon_o}\{Q_{enclosed}\} = \frac{1}{\epsilon_o}\left\{\rho \frac{4}{3}\pi r^3\right\}$$

$$E = \frac{1}{4\pi r^2 \epsilon_o}\left\{\rho \frac{4}{3}\pi r^3\right\} = \frac{\rho r}{3\epsilon_o}$$

Using $\rho \frac{4}{3}\pi R^3 = Q$

$$E(r < R) = \frac{Qr}{4\pi R^3 \epsilon_o}$$

5.b. Explain Lenz's law and mention its significance.

According to Lenz's law "the direction of induced EMF is always in such way that in creates an internal magnetic field in the loop, known as induced magnetic field so as to compensate any change that happens in the associated magnetic field of this loop". This induced EMF is always in opposition to the cause that creates this change in magnetic field, hence,

$$\varepsilon = -\frac{d\emptyset_B}{dt}$$

The negative sign indicates the opposition to induced EMF. This total expression is generally called the Faraday's law.

In integral form it can be expressed as

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\emptyset_B}{dt}$$

Where $\vec{E}$ is the electric field inside this loop at tiny current element $\overrightarrow{dl}$

In differential form the same equation can be written as

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Where B is the magnetic field at the point of element dl considered above.

- It infers the direction of induced current that is always opposing the change that caused it.
- It is crucial for upholding the fundamental Law of Conservation of Energy.
- It explains the working principle behind many electrical devices like generators and inductors.

6. a. Derive an expression for electromagnetic wave equation in free space using the Maxwell's equations.

Consider the Maxwell's equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Substituting $D = \epsilon E$ and $B = \mu H$ in the above equations we get

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

To derive wave equation in terms of electric field, the term $\vec{H}$ has to be eliminated. Taking curl on both sides the equation 5.3 we get

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} \left(\nabla \times \vec{H} \right)$$

According to vector analysis $A \times (B \times C) = B(A.C) - C(A.B)$. Thus

$$\nabla \times \nabla \times \vec{E} = \nabla \left(\nabla . \vec{E} \right) - \nabla^2 \vec{E}$$

As per the Maxwells equation $\nabla . \vec{D} = \rho_v$. Since $D = \epsilon E$ it could be written as $\nabla . \vec{E} = \frac{\rho_v}{\epsilon}$. Substituting in the above equation we get

$$\nabla \times \nabla \times \vec{E} = \nabla \left(\frac{\rho_v}{\epsilon} \right) - \nabla^2 \vec{E}$$

Substituting equation 5.6 in equation 5.5 we get

$$\nabla \left(\frac{\rho_v}{\epsilon} \right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} \left(\nabla \times \vec{H} \right)$$

Substituting equation 5.4 in 5.7 we have

$$\nabla \left(\frac{\rho_v}{\epsilon} \right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

the above equation could be rewritten as

$$\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu \frac{\partial \vec{j}}{\partial t} + \nabla \left(\frac{\rho_v}{\epsilon} \right)$$

The LHS in equation 5.9 represents a propagating wave and the RHS the source of origin of the wave. Here μ and ϵ are respectively Absolute permeability and Absolute permittivity of isotropic homogeneous medium. In case of propagation of EM wave in free space ($\vec{J} = 0, \rho_v = 0$) equation 5.9 reduces to

$$\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

6.b. State (i) Biot-Savart's law and (ii) Ampere'ss law

Biot-Savart Law

It is a fundamental law in electromagnetism that describes the **magnetic field produced by a steady electric current**. It provides a way to calculate the magnetic field (B) at a specific point in space due to a small segment of current-carrying wire.

It is analogous to Coulomb's Law in electrostatics, which describes the electric field produced by a static electric charge.

The Biot-Savart Law states that the magnetic field dB at a point P due to a small current element Idl is:

- Directly proportional to the current I.
- Directly proportional to the length of the current element dl.
- Directly proportional to the sine of the angle (θ) between the current element dl and the position vector r (from the element to the point P).
- Inversely proportional to the square of the distance r from the current element to the point P.
- Its direction is perpendicular to both dl and r, given by the right-hand rule.

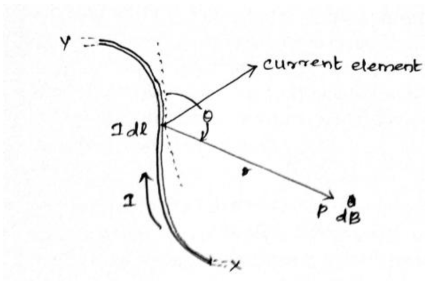


Illustration of Biot-Savart law

The magnetic field dB due to a current element Idl is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I(d\vec{l} \times \vec{r})}{r^3}$$

Where:

- dB is the differential magnetic field vector at point P.
- μ_o is the permeability of free space, a fundamental constant ($\mu0=4\pi\times10^{-7}$ T·m/A).
- I is the current in the wire (in Amperes).
- dl is a vector representing the infinitesimal length of the current element in the direction of the current flow.
- r is the position vector from the current element dl to the point P where the magnetic field is being calculated.
- r is the magnitude of the position vector r (the distance from dl to P).
- The term (dl×r) denotes the **cross product** of the current element vector and the position vector, which determines the direction of dB.

.Ampere's Law

Ampere's Law is a fundamental principle in electromagnetism that relates the **magnetic field around a closed loop to the electric current passing through that loop**. It is one of Maxwell's equations and is particularly useful for calculating magnetic fields in situations with high symmetry.

"Ampere's Law states that **the line integral of the magnetic field** (B) **around any closed loop is directly proportional to the total electric current** (Ienc) **passing through the area enclosed by that loop."**

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

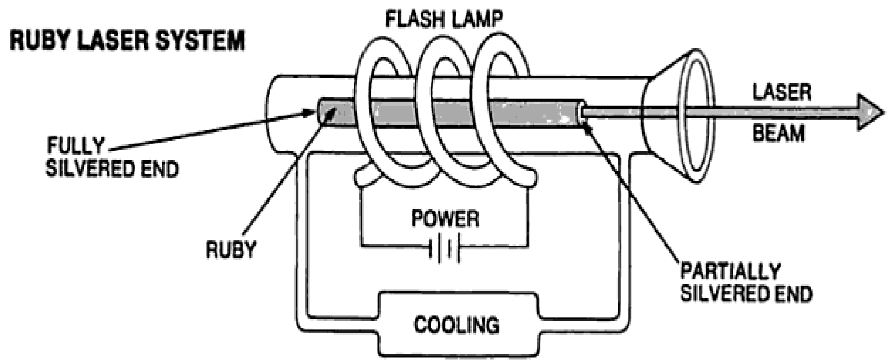
7. a. Explain the construction and working of Ruby Laser with neat sketch.

Ruby Laser

- It is the first solid state three level laser invented by T H Maiman in the year 1960.
- It produces a high output power in the order of Mega Watts with ten nanoseconds pulse duration.
- Pumping Method: Optical Pumping Scheme.
- Laser Active Medium : Chromium atoms

Construction

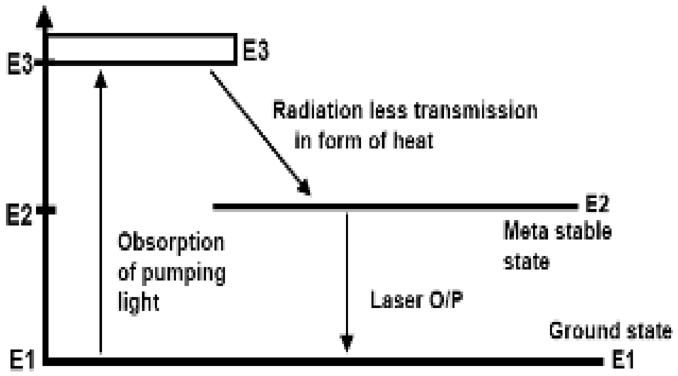
1. It consists of a long cylindrical ruby rod which is made up of Aluminum Oxide (Al_2O_3) and is doped with 0.05% of Cr_2O_3.
2. Due to the presence of Cr^{3+} ions the ruby rod is appears as pink in colour. This suggest that there is a strong absorption in the visible region.
3. The length of the ruby rod is about 10cm and diameter is 0.5cm.
4. The end faces of the rod are grounded and polished such that the end faces are exactly parallel to each other.
5. One of the ends is silvered for 100% reflection and the another end is silvered for nearly 90% reflection.
6. A helical Xenon flash lamp is surround the ruby rod and is connected to a high voltage trigger pulse (~20kV).
7. During the process, a large amount of heat is produced. So, the system is cooled with the help of a coolant (water) circulating around the ruby rod.



Working

1. Laser action will be takes place between the energy levels of the Cr^{3+} ions. Al_2O_3 is an insulator and the introduction of Cr^{3+} ions results in additional energy levels within the band gap.

2. The Chromium ions having three active energy levels known as E1 - Ground state, E2- Meta stable state and E3- Higher excited state.



3. E3 state is fairly wide and hence can accept a wide range of wavelengths. It has short life time.
4. In this the lasing action occurs between E2 and E1.
5. When ruby rod is irradiated with flash lamp, Chromium ions absorb the light of wavelength at around 5600A° (5000-6000A°) which will be either green or blue color.
6. As a result the ions transferred to higher excited state E3 from Ground state E1.
7. From this level (E3) the ions will go down to Meta stable state (E2) in a non-radiative transition. This energy is transferred to the crystal vibrations and changed into heat.
8. Since the life time of the E2 level is in the order of milliseconds, Chromium ions remain in this level for longer duration.
9. So population inversion takes place between Meta stable state (E2) and Ground State (E1).
10. The spontaneously emitted initial photons would travel in all the directions, of these, those travelling parallel to the axis of the rod would be reflected at the ends and pass many times through the amplifying medium and stimulate the atoms in Meta Stable state.
11. As a result stimulated emission takes place and chromium ions translate from E2 to E1.
12. This transition gives rise to the emission of light of wavelength 6943A°.
13. The output of this laser consists of a series of laser pulses for duration of milliseconds or less and the diameter of the beam is 1 mm to 25mm.

7. <u>b. List four characteristics of Lasers</u>

Lasers have several unique characteristics that set them apart from ordinary light sources:

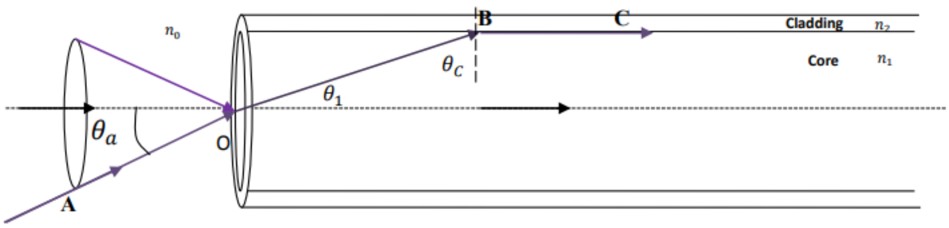
1. **Monochromaticity:** Laser light consists of a single, highly pure color (wavelength). Unlike white light which contains a spectrum of colors, laser light has a very narrow range of wavelengths.
2. **Coherence:**
   - **Spatial Coherence:** All the light waves are in phase with each other across the entire beam. This allows lasers to be focused to a very small spot.
   - **Temporal Coherence:** The light waves maintain a constant phase relationship over a long distance or duration. This makes them useful for applications like holography.

3. **Directionality (Low Divergence):** Laser light travels in a very narrow, concentrated beam with minimal spreading over long distances. This is why a laser pointer beam remains tight over many meters.
4. **High Intensity/Brightness:** Because laser light is highly directional and coherent, a large amount of energy can be concentrated into a very small area, making lasers extremely bright and intense.

8. <u>a. Build an equation for acceptance angle and numerical aperture of an optical fibre.</u>

Expression for Numerical aperture and Condition for propagation:

Consider a ray of light in a medium of RI 'n_0' entering in to a fiber having a core of RI 'n_1' and cladding of RI 'n_2' at a point "O" on the core surface. The ray OA incident at O, at an angle θ_a refracts in to the core at an angle θ_1 and falls on the core-cladding interface at an angleθ_C at B and grazes the interface along BC after refraction.



For the refraction at 'O', Snell's law can be written as $\boldsymbol{n_0 \sin\theta_a = n_1 \sin\theta_1}$

$$\therefore \boldsymbol{\sin\theta_a = \left(\frac{n_1}{n_0}\right)\sin\theta_1} \quad \text{.........(1)}$$

Similarly For the refraction at 'B', Snell's law becomes $\boldsymbol{n_1 \sin\theta_C = n_2 \sin 90°}$

But we have $\boldsymbol{\theta_C = (90° - \theta_1)}$ $\quad\therefore\ \boldsymbol{n_1 \sin(90° - \theta_1) = n_2}$

$$\text{Or } \boldsymbol{\cos\theta_1 = \left(\frac{n_2}{n_1}\right)}$$

$$\Rightarrow \boldsymbol{\sin\theta_1 = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} == \sqrt{\frac{(n_1^2 - n_2^2)}{n_1^2}}} \quad \text{........ (2)}$$

$$\therefore \text{Eqn. (1) becomes } \boldsymbol{\sin\theta_a = \left(\frac{n_1}{n_0}\right) \cdot \sqrt{\frac{(n_1^2 - n_2^2)}{n_1^2}}}$$

$$\Rightarrow \text{N. A.} = \boldsymbol{\sin\theta_a = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}}$$ (This is the expression for the Numerical aperture.)

If the surrounding medium is air then N . A . $=\boldsymbol{\sin\theta_a = \sqrt{(n_1^2 - n_2^2)}}$ .... (3)

8. b. Distinguish between step index and graded index optical fiber.

Step index fiber	Graded index fiber
1. In step index fibers the refractive index of the core medium is uniform through and undergoes an abrupt change at the interface of core and cladding.	1. In graded index fibers, the refractive index of the core medium is varying in the parabolic manner such that the maximum refractive index is present at the center of the core.
2. The diameter of core is about 10micrometers in case of single mode fiber and 50 to 200 micrometers in multi mode fiber.	2. The diameter of the core is about 50 micro meters.
3. The transmitted optical signal will cross the fiber axis during every reflection at the core cladding boundary.	3. The transmitted optical signal will never cross the fiber axis at any time.
4. The shape of propagation of the optical signal is in zigzag manner.	4. The shape of propagation of the optical signal appears in the helical or spiral manner
5. Attenuation is more for multi mode step index fibers but Attenuation is less in single mode step index fibers	5. Attenuation is very less in graded index fibers
6. Numerical aperture is more for multi mode step index fibers but it is less in single mode step index fibers	6. Numerical aperture is less in graded index fibers

9.a. Derive Schrodinger Time-independent wave equation

Consider a particle of mass *m* is moving with velocity *v* along positive x- axis direction. Since moving particle is associated with matter waves of wavelength λ. The wave function is denoted by ψ.

Let ψ(x,t) is wave function of a wave moving along x – direction.

1-D wave equation is $\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$

$\psi(x,t)=\psi_o(x)e^{-i\omega t}$ -----------------1

Differentiating equation w.r.t t

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_o(x) e^{-i\omega t} = -\omega^2 \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = -\frac{1}{v^2}\left(2\pi \frac{v}{\lambda}\right)^2 \psi$$

$$==$$

$$= -4\pi^2 \left(\frac{1}{\lambda}\right)^2 \psi = -\frac{4\pi^2}{h^2} p^2 \psi$$

$= -\left(\frac{4\pi^2}{h^2}\right) 2m(E-V)\psi$ Since $\frac{p^2}{2m} + V = E$

Therefore $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} == -\frac{8\pi^2}{h^2} m(E-V)\psi$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2}{h^2} m(E-V)\psi = 0 \text{ -------------(2)}$$

This is called Schrodinger time independent wave equation in 1-D

Substituting equation 1 in equation2 we get,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}\left(i\hbar \frac{\partial}{\partial t} - V\right)\psi = 0$$

Rearranging the terms

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}\left(i\hbar \frac{\partial}{\partial t} - V\right)\psi = 0$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2}\right) + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

This is called Schrodinger time independent wave equation in 1-D

$$\left(-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\right) + V\right)\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$E\,\psi = H\,\psi$$

H=$\left(-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\right) + V\right)$ is called Hamiltonian operator

In 3-D

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$= \nabla^2 \psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

Where $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$ is called Laplace operator.

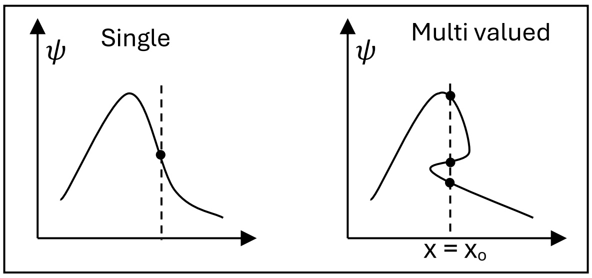
9.b. Outline the significance of wave function

Wave function must satisfy the following criteria in order to be a wave function. It must be a well behaved mathematical function.

1. ψ must be finite
2. ψ must be single valued
3. ψ and its first order space derivatives must exist and be continuous.
4. ψ must be square integrable and normalized.

Why ψ must obey the above conditions? Because, if ψ is not finite then the probability of finding of the system in state ψ will have a value infinity which is meaningless.

ψ must be single valued because, it should give only one probability, but not many values, at a given time and position.



ψ must be continuous because from whichever direction you approach a point $x = x_o$, the value for probability should not change. If it is discontinuous it will give two different answers for probability at a given point at a given time.

ψ must be square integrable, means,

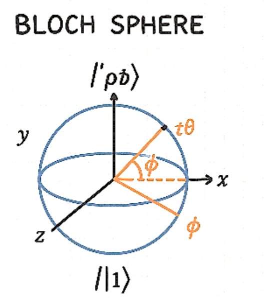
$$\int_{-\infty}^{+\infty} |\psi|^2 dV < \infty$$

ψ must be Normalized, means,

$$\int_{-\infty}^{+\infty} |\psi|^2 dV = 1$$

This condition is known as Max Born's probability normalization condition which simply states that the total probability of finding a system is always unity.

10.a. Explain Bloch sphere and Entanglement in Quantum computing.



Bloch sphere:

The Bloch Sphere is a geometric representation of a **qubit**, the basic unit of quantum information. It shows all possible pure states of a qubit as points on the surface of a sphere.

A general qubit state can be written as:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

Where:

- $\theta \in [0, \pi]$ is the polar angle
- $\phi \in [0, 2\pi]$ is the azimuthal angle
- $|0\rangle$ and $|1\rangle$ are computational basis states

State Representation

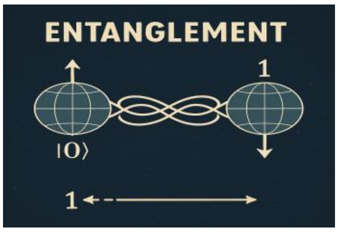
Any qubit state:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

- $|0\rangle$: Located at the **north pole** of the sphere ($\theta = 0$)
- $|1\rangle$: Located at the **south pole** of the sphere ($\theta = \pi$)
- **Superpositions**: Lie somewhere else on the sphere depending on θ and ϕ
- **Equator**: Represents equal probabilities of measuring $|0\rangle$ or $|1\rangle$, like $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Entanglement:

- Entanglement is a fundamental principle of quantum mechanics where two or more qubits become interconnected with each other.
- So that the state of one qubit instantly influences the state of another, regardless of the distance between them.
- This phenomenon allows quantum computers to process and share information in ways that are impossible for classical systems.
- In quantum computing, entanglement is crucial for enabling powerful operations like quantum parallelism and quantum teleportation.
- It is widely used in algorithms (e.g., Shor's and Grover's), error correction, and secure communication protocols such as quantum key distribution.
- By harnessing entanglement, quantum systems achieve exponential speedup over classical counterparts for specific tasks.



Example 1 of Quantum Entanglement:

Imagine two entangled qubits, A and B, created in such a way that their states are perfectly correlated—if A is measured as 0, B will be 1; if A is 1, B will be 0. These qubits are then separated by a large distance.

When qubit A is measured and found to be 0, qubit B instantly becomes 1—even if it's on the other side of the universe. This instantaneous correlation occurs without any signal traveling between them, illustrating the "spooky action at a distance" Einstein famously referred to.

10. b. Distinguish between qubits and classical bits

S.No.	Bits	Quantum Bits
1.	A Bit, also called Binary Digit or Classical Bit, is the smallest unit of information measurement in digital computing technology.	A Quantum Bit, also called Qubit, is the smallest unit of information measurement in quantum computing.
2.	A bit can have only two values, i.e. 0 and 1.	A quantum bit can have multiple values simultaneously.
3.	Classical bit does not follow superposition property.	Quantum bit follows superposition property.
4.	Bits are inherently stable, i.e. they do not change their states in the absence of external force.	Quantum bits are inherently unstable, i.e. they can change their states even no external force exists.
5.	The value or state of a bit can be determined precisely. Hence, they are deterministic.	The value or state of a quantum bit cannot be precisely determined. Hence, they are probabilistic.
6.	Bits are physically implemented through electronic and optical devices.	Quantum bits are implemented by using quantum systems like ions, atoms, superconductors, etc.
7.	Boolean operations are executed on bits.	Quantum operations are executed on quantum bits.
8.	Bits can be copied perfectly.	Quantum bits cannot be copied perfectly.
9.	The operations on bits are performed using digital logic gates, such as AND, OR, NOT, etc.	The operations on quantum bits are performed using quantum logic gates.